

Evaluation of Takeoff Performance Monitoring Algorithm in Large Jet Transport Operations

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The issue of monitoring aircraft performance during takeoff has been repeatedly considered in the last 50 years. This has been in response to concern over the low safety margins associated with this phase of flight and the lack of adequate objective performance-related information available to the crew during the takeoff maneuver. Two major difficulties, however, have to date inhibited the successful introduction of an instrument providing such information on the flight deck, namely, the provision of a reliable quantification of actual performance during the takeoff maneuver and the provision of an adequate display. This paper focuses on the former issue and evaluates the accuracy of a predictive performance monitoring algorithm developed at Cranfield University. The major hurdles that are associated with takeoff performance monitoring are briefly discussed, and the results of flight-based tests using data from large jet-transport operations presented. These results, which confirm a robustness that warrants the application of the algorithm in real-time performance monitoring, are also compared with those of an earlier flight test program using a twin turboprop aircraft.

Nomenclature

a	=	acceleration
a_0, a_1, a_2	=	acceleration function polynomial coefficients
C_D	=	coefficient of drag
C_L	=	coefficient of lift
m	=	mass
S	=	wing surface area
T	=	thrust
V_g	=	ground speed
v	=	airspeed
W	=	aircraft weight
$\alpha_0, \alpha_1, \alpha_2$	=	thrust function polynomial coefficients
θ	=	runway slope
μ	=	coefficient of rolling friction
v_w	=	wind speed (headwind component)
ρ	=	air density

Introduction

THE takeoff maneuver is considered to be a crucial phase of flight, particularly in terms of safety and mission viability. This is because aircraft are regularly required to operate with as high a load as possible in demanding situations such as high airfield altitude and temperature and in adverse weather conditions. These factors all result in increased runway length requirements to ensure that the takeoff attempt will be completed successfully. A takeoff attempt is, in this text, defined as successful if the maneuver is completed without incident and unsuccessful otherwise. In the limiting condition, this definition is interpreted to imply that actual runway usage does not exceed the allowance calculated before takeoff in successful maneuvers. As the runway length available is limited, it is often a restricting factor, and there is a risk associated with the eventuality of overrunning and striking obstacles at the end of the runway. Therefore, a compromise exists between the maximum

dispatch weight of the aircraft in the particular operating conditions and the risk of overrunning, which constitutes an accident in commercial aviation terms.

The mechanism of protection adopted in all sectors of aviation in which takeoff is not conducted vertically is that of scheduled performance. In the takeoff phase, this involves the estimation of the runway length required to complete the maneuver successfully and ensuring that the runway length required is available. The aircraft is then dispatched, and during the actual takeoff the runway length should prove adequate to allow the aircraft to complete the takeoff successfully. This, however, is not always the case, with recent statistics indicating that takeoff-related accidents contribute to approximately 14% of all accidents,¹ some of which are caused by the aircraft exceeding the runway distance available. However, scheduled performance calculations are associated with the prediction of the distance that will be required, and an uncertainty is associated with those calculations. As the distance required approaches the distance available, the risk of exceeding the available distance increases. It is on this premise that scheduled performance is based, allowing for a reasonable probability that the aircraft will complete the takeoff maneuver successfully. The contingencies and detail catered for in scheduled performance are epitomized by large, civil-transport aircraft operations under FAR/JAR Part 25. This class of operations demands the highest levels of safety and, accordingly, imposes the highest restrictions in performance of all categories of aircraft operation.

The origins of scheduled performance date back to the early 1950s, and the method is effectively based on the target of reducing the probability of failing to achieve obstacle clearance on takeoff to less than 1×10^{-7} . Such an event is considered to be a hazardous incident by the regulatory bodies, and the eventuality should be extremely remote (Fig. 1). The mechanism with which this probability is ensured involves the inclusion of calculated leeway to the expected runway distance requirement during performance scheduling. To this effect, three eventualities during takeoff are considered, namely, 1) the “normal” situation where the takeoff is uneventful, 2) the situation in which an engine failure occurs requiring the run to be aborted, and 3) the situation in which the engine failure occurs late in the run and the takeoff will have to be continued on the remaining thrust available.

In the engine-failure cases, the most demanding situation in terms of runway length requirement is when the failure occurs at the decision speed V_1 . The engine failure rate used in scheduled performance is 1×10^{-6} per engine second at takeoff power,³ and the probability of a two-engine aircraft experiencing an engine failure in 1 s about V_1 is therefore approximately 2×10^{-6} . Adding the situation that

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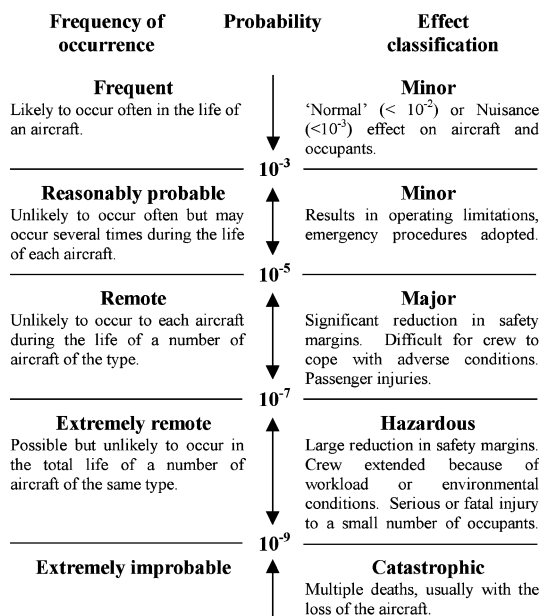


Fig. 1 JAA-defined relationship between probability and severity of failure condition (reconstructed from AMJ25.1309²; probabilities given per hour).

only 10% of takeoffs are expected to be conducted in field-limiting conditions and the fact that the estimated runway distance required statistically implies a 50% probability of exceeding this distance, the overall probability of exceeding the runway allowance is estimated to be of the order of 1×10^{-7} .

Protection in the normal condition is achieved by estimating the effect of the variations that can be expected in the performance variables (e.g., ambient temperature and pressure, aircraft weight, and wind-speed component) on the runway length requirement. Detailed analysis has resulted in the distance requirement being characterized by a normal distribution with a standard deviation of 3% about the expected length.³ An allowance of five standard deviations, or 15%, will, therefore, introduce a leeway sufficient to ensure that, statistically, only about 1 in 10^7 takeoffs exceed this limit. Besides this leeway, scheduled performance also introduces other arbitrary allowances to ensure successful completion of the run, such as conservatively taking into account only half of the expected value of headwind in the calculation and allowing for engine-failure recognition times.

When analyzing the merits of scheduled performance in the context of operations, two issues become evident. The first is the fact that scheduled performance only provides a statistical means of protection during takeoff. Consequently, protection focuses more on the global safety record than on the safety of a particular run. This has resulted in a process that is effectively open loop. Indeed, from a performance point of view, once runway length requirements are estimated prior to dispatch, there is no provision for any objective assessment of whether the actual performance of the aircraft during the maneuver is adequate or otherwise. Such means, in the form of an instrument referred to as a takeoff performance monitor, would effectively close the loop and significantly reduce the risk of overrun through the refocusing on the individual takeoff maneuver (Fig. 2).

The second issue, which becomes evident on closer analysis, is that scheduled performance protects only against normal operating conditions. In the engine-failure conditions, for example, it is assumed that performance has been normal (average) up to the point of engine failure. Furthermore, following the failure, conditions are assumed to remain normal (that is, only the one, contained, and well-defined engine failure has occurred) in both the continued and rejected takeoff case. This implies that all systems are assumed to continue functioning well and no event that might compromise aircraft performance beyond the consideration of the predispatch cal-

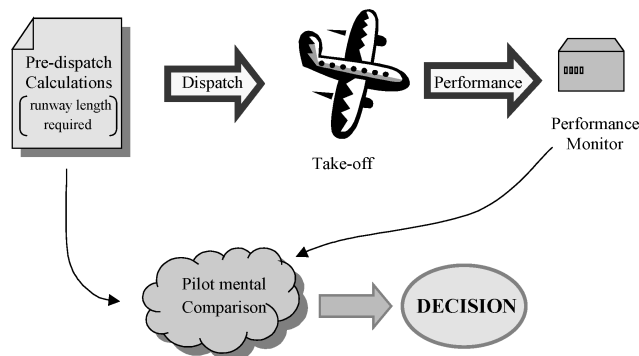


Fig. 2 Role of the takeoff performance monitor in the decision to continue or reject the run.

culations will occur. Likewise, the all-engines-operative case only includes allowances for normal variations between the estimated and actual dispatch conditions. It does not, for example, protect against tire bursts or combinations of failures. Neither does it protect against pilot error (such as misset thrust or violation of the V_1 principle), aircraft misloading, or, indeed, against errors in the actual calculations.

Whereas the first issue has been the main argument that has been used to back the role of the takeoff performance monitor, the second has mostly been overlooked. This situation is aggravated by the fact that when anomalies that are not protected by scheduled performance do occur they do not usually occur simultaneously with contingencies allowed for by scheduled performance, and the "unused" leeway allowed for the latter will often prove adequate to cater for the former occurrence. As a result, the incident might go unnoticed. This, however, is not always the case. Indeed, a close inspection of runway overruns indicates that on many occasions the primary cause would have been one that is not protected against by scheduled performance and current practice. Of greater significance is the fact that a number of these accidents can be considered avoidable, particularly if information regarding aircraft performance were available to the crew during the takeoff run.

Cranfield Takeoff Performance Monitor

The preceding discussion forms the basis of the Cranfield design methodology and has led to the conclusion that the takeoff performance monitor would be most useful if it could provide precisely that information that is critical to the decision to continue or reject the run but is currently not available to the crew. The approach, therefore, has been one of improving the effectiveness of current procedure and scheduled performance and not to replace them. This approach also facilitates certification and acceptance of the proposed instrument by the industry. As current technologies do not support the reliable estimate of the implications of certain contingencies such as multiple tire bursts, the Cranfield program has identified the most significant improvement to safety to be the identification of subtle underperformance that would otherwise be difficult for the crew to detect during the actual takeoff run. By providing performance information in an appropriate and ergonomic fashion, the crew would be in a better position to judge whether or not it would be more advantageous to continue the takeoff or otherwise. While critically indicating substandard performance, the instrument would further give the crew confidence when the performance is adequate and the takeoff is progressing normally.

To fully exploit the concept of performance monitoring, a system should be capable of forward-predicting the vehicle's performance. It has long been accepted that an indication of the current situation during the run (for example, by measuring current position in relation to achieved airspeed or by indicating current acceleration) does not reliably quantify the overall performance of the takeoff in terms of the actual runway length that will eventually be needed to complete the run. Predictive systems, capable of estimating and

indicating this quantity, are therefore favored. Although it is implicit that any prediction is invariably based on the assumption that the parameters measured up to that moment in real time will remain unchanged, this is considered acceptable for the purposes of takeoff performance monitoring.

The takeoff is traditionally divided into several phases. The acceleration phase, describing the initial run up to rotation speed V_R , is followed by the rotation phase that extends to the point of liftoff. The airborne part of the takeoff, ending at the screen height, 35 ft above the extended runway surface, is referred to as the transition phase. In the case of the rejected run, the deceleration phase is assumed to start at the moment the run is aborted, normally initiated by the reduction of thrust and thus includes the period during which the retarding mechanisms are activated and ends at the moment the aircraft comes to rest.

A detailed consideration of the nature of each phase of the takeoff maneuver will quickly result in the appreciation that post- V_R distances cannot be predicted with a reliability that is adequate for the purposes of takeoff performance monitoring.⁴ This is because the instant of rotation, the actual rotation, and the transition distances are highly dependent on piloting technique and introduce such a large uncertainty that a prediction is effectively relegated to an estimate with an accuracy comparable with that of scheduled performance. Similar difficulties exist in the estimation of the deceleration distance. The Cranfield design team has concluded that it is best to allow scheduled performance for any post- V_R distance requirements. Scheduled performance introduces the statistically necessary and industry-accepted allowances to ensure the level of required safety, and the use of these distances, therefore, will be acceptable to both the authorities and operators.

Because the best method of monitoring takeoff performance involves the estimation of the runway distance required and comparing this estimate with scheduled performance, the Cranfield approach effectively reduces the task of real-time performance prediction to estimating the distance the aircraft will cover to V_1 , which, because V_1 cannot exceed V_R , is predictable. This results in a conservative estimate, because in situations where performance is adequate more runway than scheduled would be still available, whereas in cases where performance is substandard the system would give early warning to allow the run to be aborted well before V_1 and accordingly increasing the probability of success in the intended maneuver.

Equations of Motion

In the ground run up to V_1 , the aircraft is essentially traveling straight and level and, for the purpose of performance calculations, can be considered to have only one degree of freedom as the motion of the aircraft is essentially in a straight line along the runway centerline. The universally adopted equation describing the vehicle's motion is accordingly derived from the application of Newton's law on a lumped parameter model, resulting in the acceleration of the aircraft being described by Eq. (1).

$$a(V_g) = \left\{ T - \left[\frac{1}{2} \rho S (C_D + \mu C_L) \right] (V_g + v_w)^2 - W [\sin \theta + \mu \cos \theta] \right\} / m \quad (1)$$

where the thrust T is usually expressed as a second-order polynomial function of airspeed:

$$T = \alpha_2 (V_g + v_w)^2 + \alpha_1 (V_g + v_w) + \alpha_0 \quad (2)$$

The acceleration of the aircraft up to V_1 is consequently modeled as a second-order polynomial of airspeed:

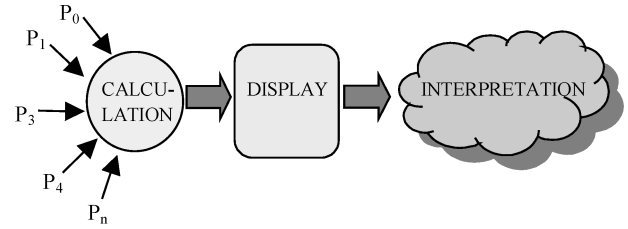
$$a(v) = a_0 + a_1 v + a_2 v^2 \quad (3)$$

This basic equation is universally used to obtain a function expressing the distance-to-go to V_1 , which is then added to the distance gone to determine the overall distance required to the decision speed. The distance-to-go to V_1 has, in past designs, been calculated in different ways and in certain instances is proprietary. The Cranfield design

Table 1 Summary of the estimated accuracy of the prediction algorithm during Jetstream-100 flight testing^a

Maximum error	Stage 2, %	Stage 3, %
Average	2.26	1.20
Standard deviation	1.11	0.56
99% conf.	4.83	2.49

^aResults are presented in terms of percentage of actual distance to V_1 .



KEY: $P_n = n^{\text{th}}$ parameter used in the performance calculations.

Fig. 3 Information chain in takeoff performance monitoring.

likewise derives the estimated distance-to-go from the preceding expressions and utilizes a smart technique to obtain high reliability in the estimate.

Algorithm Accuracy

It is immediately appreciated that system reliability is a fundamental requirement in takeoff performance monitoring. Low reliability would reduce confidence in the system, result in the failure to detect substandard performance, and also introduce unnecessary takeoff rejections, thereby defeating the instrument's purpose by inadvertently increasing the risk of accident. Reliability in takeoff performance monitoring should be assessed in terms of how well the system facilitates the crew's correct decision to continue or reject a run and their capability of completing the maneuver successfully. This clearly renders critical the ergonomic design of the display. The information chain presented in Fig. 3, however, also illustrates the importance of the calculation's timely and correct (accurate) estimate of actual aircraft performance and imposes stringent requirements on both the concept of the algorithm (for example, whether it is predictive, what parameter representing performance is calculated, etc.) and the actual accuracy of the result.

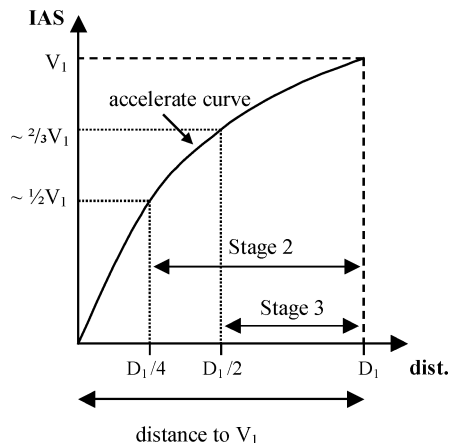
The scope of the operational evaluation presented in this work is specifically to confirm the accuracy of the algorithms used in a takeoff performance monitor developed at Cranfield University.

Algorithm Validation Program

The Cranfield development program has included an extensive algorithm validation program that has been based on flight tests using the university's Jetstream-100 flying laboratory, with simulator testing only adopted to assess the algorithm's performance in hazardous conditions.⁵ The results of the flight tests⁶ are summarized in Table 1, where the estimated accuracy of the prediction is presented in terms of the percentage of the distance to V_1 . For the purposes of algorithm performance quantification, the run to V_1 has been divided into three stages. The first stage comprises all of the run to V_1 . In the second stage, a quarter of the distance to V_1 will have been passed, and the algorithm will therefore have to predict ahead less than 75% of the run to V_1 . In the third stage the forward prediction will progressively fall from 50% of the distance to V_1 to no forward prediction at all (Fig. 4). This segmentation effectively allows the determination of the maximum algorithm prediction error, in terms of distance, experienced (and expected) in the last 75% and last 50% of the run to V_1 . The quarter-and-half distance to V_1 were chosen as boundaries because, as the aircraft transits these points, the crew will still have approximately one-half and one-third of the total time to V_1 respectively to act on any decision before V_1 is transited.

Table 2 Summary of the salient performance parameters of the B747-400 takeoffs

Value	Weight, lb	V_{gROT} , kn	Run time to V_R , s	Distance to V_R , m
Minimum	462,720	114	20	660
Average	724,130	144	37	1518
Maximum	874,240	173	53	2352

**Fig. 4** Segmentation of the run to V_1 for the purpose of quantifying system accuracy.

The results of the flight tests indicated that the algorithm is capable of predicting runway requirements to an accuracy that is acceptable for the reliable monitoring of aircraft performance in real time.^{7,8} The performance standards for takeoff monitoring systems have been set by the Society of Automotive Engineers⁷ at 5% of the apparent all-engines-operating takeoff distance throughout the takeoff run, and a modified standard has been proposed by Cranfield University⁸ in which the accuracy improves from 5% error at quarter distance to V_1 toward zero error at V_1 . Both standards require this accuracy on 99% of all runs.

The figures presented in Table 1 clearly warranted the extension of the validation of the test program to assess the algorithm's performance on aircraft more typical of commercial transport operations. To this effect, the algorithm, adapted to model the relevant aircraft, was tested using data recordings of Boeing B747-400 aircraft on normal commercial operations. Forty flight recordings of takeoffs were arbitrarily selected, covering a reasonable range of operational conditions, including various combinations of thrust, weight, and departure runway (Table 2). The takeoffs also had varying degrees of rolling start. The start of the maneuver had therefore to be defined as the instant when the thrust levers are advanced beyond the intermediate position when the crew would be carrying out the relevant engine checks. The large variation in the run time to rotation indicates the effects of these different combinations. The final (target) velocity was taken to be rotation speed V_R , which in practice is the upper limit of V_1 .

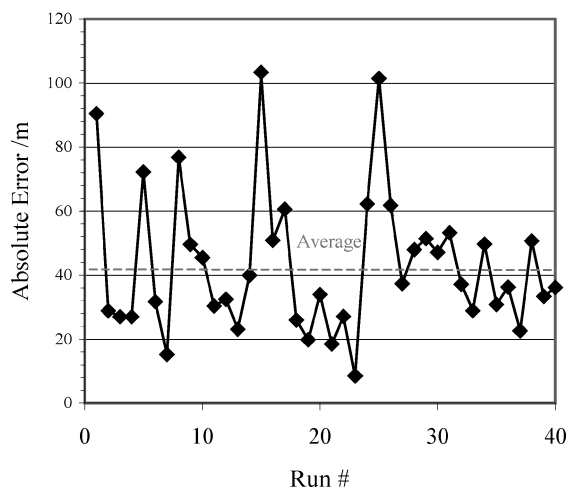
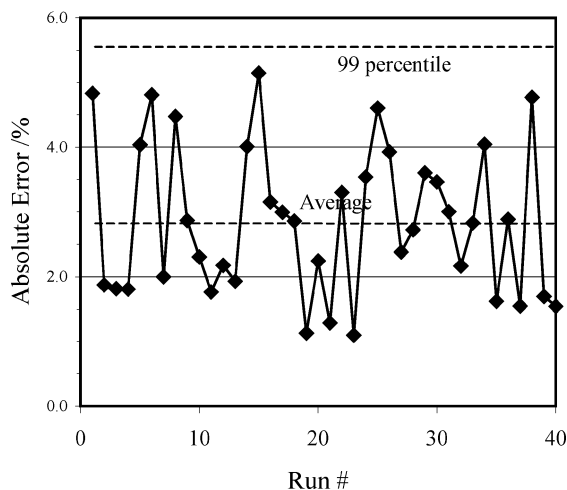
Performance Results and Analysis

The algorithm was run on the 40 data sets, and the error of the estimate was recorded in conjunction with the time elapsed and distance gone. The algorithm updates its prediction 10 times a second; this update rate has been justified in earlier works and proved to be suitable for the Cranfield monitor design. All predictions exhibited a decreasing error as the decision speed is approached; this was expected because the algorithm will be required to progressively look less far ahead as the run progresses. Indeed, at V_1 , the algorithm only needs to determine the current position, and this reduces the uncertainty to that associated with the calculation of the distance covered. Although the graphical presentation of the prediction error plotted as a function of time or distance provides a very good insight into the qualities of the prediction on specific flights, the

Table 3 Summary of the estimated accuracy of the prediction algorithm using the B747-400 data recordings^a

Maximum error	Stage 2		Stage 3	
Average	43.2 m	2.86%	17.5 m	1.15%
Standard deviation	—	1.15%	—	0.61%
99% conf.	—	5.54%	—	2.57%

^aResults are presented in absolute terms and in terms of percentage of actual distance to rotation from the start of the run.

**Fig. 5a** Maximum algorithm prediction error in stage 2 of the run, presented in terms of distance.**Fig. 5b** Maximum algorithm prediction error in stage 2 of the run, presented as a percentage of the total distance covered to V_1 .

overall prediction capabilities of the algorithm need to be quantified in terms of a maximum error with a particular confidence level. To this effect, the algorithm performance has been quantified by determining the maximum absolute error experienced in the last two stages of each run. Positive and negative prediction errors lead to overestimating and underestimating the distance requirements, respectively. Both types are considered to be equally detrimental to the confidence in the algorithm calculation, and therefore the absolute value of the maximum error was considered relevant to the analysis. These results, which document the worst performance estimate actually calculated in each run, are presented in Figs. 5 and 6. A summary is presented in Table 3.

Table 3 indicates that during stage 2 the maximum error that would be expected is 43 m, which is slightly more than half an aircraft length (the B747-400 measures approximately 70 m in length), whereas that expected in the last stage is approximately half this value. These figures present a very conservative picture of

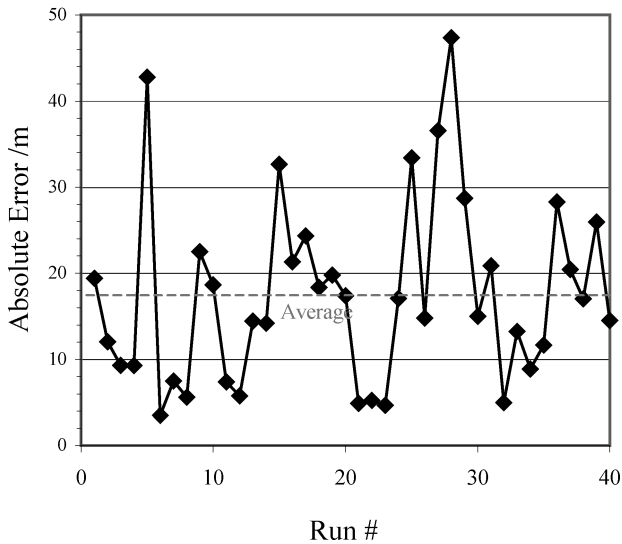


Fig. 6a Maximum algorithm prediction error in stage 3 of the run, presented in terms of distance.

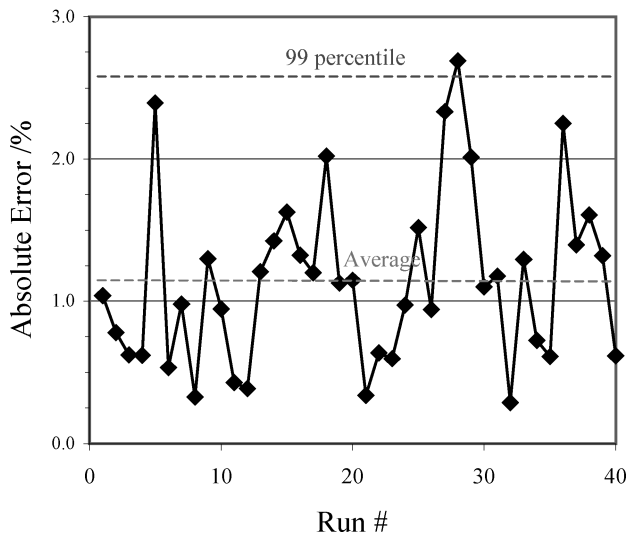


Fig. 6b Maximum algorithm prediction error in stage 3 of the run, presented as a percentage of the total distance covered to V_1 .

the qualities of the algorithm because this maximum error would in practice be experienced at the early end of the stage and the accuracy of the algorithm would be significantly better during most of the stage in question. In fact, although the maximum error in stage 2 on average is 43 m, it will be less than 18 m for two-thirds of this stage. The same effect is exhibited in stage 3.

What is of greater interest, however, is an estimate of the maximum expected error associated with a higher confidence interval to cover, for example, 99% of all runs. To obtain such an estimate, the distribution of the maximum error had to be defined. As the prediction errors are greatly dependent on variations that are random, the maximum error distribution has been approximated to a Gaussian function, even though a degree of skewness will invariably be present. This, in fact, is confirmed by the fact that the maximum error can never be negative and by the histograms of Figs. 7a and 7b.

Table 3 presents the calculated average, standard deviation and 99% confidence intervals of the maximum error with the first and last parameters being overlaid on the graphs of Figs. 5b and 6b. The standard deviation has been calculated for the percentage error but not for the absolute error because the former is a normalized error and therefore has all operating factors factored out. The calculation of the standard deviation of the absolute error would, in this case, be of no value.

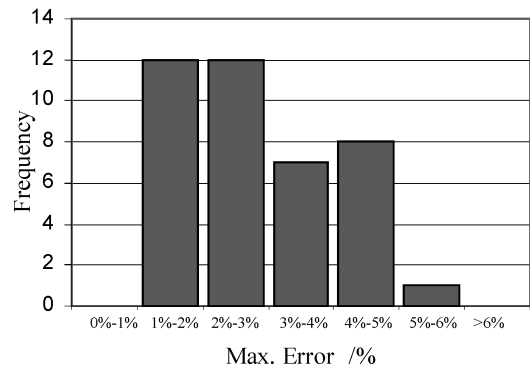


Fig. 7a Stage 2 maximum error histogram.

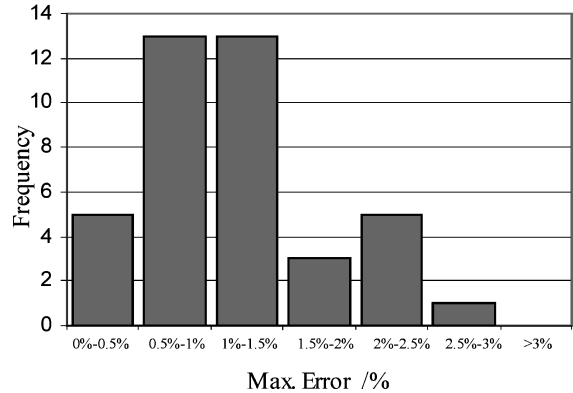


Fig. 7b Stage 3 maximum error histogram.

The skewness of the distribution functions illustrated in Figs. 7a and 7b provides an indication of the implications of the Gaussian approximation on the results presented in Table 3. Figure 7a indicates that the stage 2 error distribution is skewed to the right, and this renders the single-tailed analysis slightly conservative. Indeed, the statistics indicate that about 27% of runs should be expected to have a maximum error greater than 4% ($\mu + 1\sigma$), whereas, on observation, only 9 in the 40 runs (22.5%) exhibited such high errors. The stage 3 distribution error is likewise skewed to the right, and this indicates that the single-tailed analysis is also conservative. In fact, only 5 of the 40 runs, constituting 12.5%, were observed to have a maximum error greater than 1.76% ($\mu + 1\sigma$).

The error in the estimate has been computed by comparing the distance-to- V_1 estimate with the distance eventually covered to the decision speed, as calculated using inertial measurement sources on board the aircraft in this case; other sources providing kinematic data, such as global positioning system, could be used instead. This analysis, therefore, only takes into account the algorithm prediction errors. Specifically, it does not include errors associated with the calculation of distance gone (which include sensor/instrument uncertainties) and these have to be added to the values of Table 3 to provide an estimate of the actual total prediction errors. These latter errors are actually specific to the particular installation and instrumentation of the host aircraft and are consequently beyond the scope of this study, the primary objective of which has been the validation of the algorithm's prediction qualities. Indeed, the issue of most interest in this study has been the quantification of the modeling error, and this has been shown to be acceptably low.

Another significant source of error that has been intentionally factored out is the effect of wind gusts at V_1 . Instantaneous wind velocity cannot be predicted, and, as a result, a constant (average) value of this parameter needs to be assumed. As an indication, 1-kn error in this estimate results in approximately a 15-m prediction error, which is an uncertainty that has to be accepted. The effects of all other errors or variations in operational conditions are otherwise taken into account in the analysis. Indeed, effects such as those caused by any changes in runway gradient, changes in the wind vector, and any other dynamic condition experienced during the run are exhibited in the prediction error recorded.

Conclusions

This study has demonstrated the modeling and predictive qualities of the algorithm and indicates that the errors that can be expected during the run should be well within the requirements of Aerospace Standard AS-8044⁷ and the more stringent in-house standard developed at Cranfield University.⁸ Moreover, the value of these results is further confirmed by the close correlation with the earlier results of the Jetstream flight tests. Indeed, it can be concluded that, as the Cranfield algorithm exhibits accuracy within the accepted industry standard, it would warrant adoption into commercial operations. In addition, this analysis has further confirmed that the algorithms are applicable during rolling start takeoffs and therefore compatible with standard operations and procedures.

Acknowledgments

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